ANSWER KEY

CHAPTER 5

Similar Triangles

Exercises for Section 5.1

5.1.1 If triangles ABC and YXZ are similar, then

$$\frac{AB}{YX} = \frac{AC}{YZ} = \frac{BC}{XZ}.$$

It follows that $AB/BC = YX/XZ$ and $AC \cdot YX = AB \cdot YZ$. Hence, statements (a), (b), and (d) are true. Statements (c) and (e) are not necessarily true.

5.1.2 If triangles ABC and ADB are similar, then $AB/AD = AC/AB$, so $AB^2 = AC \cdot AD$. We are given that $AC = 4$ and $AD = 9$, so $AB^2 = 4 \cdot 9 = 36$. Therefore, $AB = \boxed{6}$.

Exercises for Section 5.2

5.2.1

(a) Since $\overline{AB} \parallel \overline{DE}$, we have $\angle A = \angle D$ and $\angle B = \angle E$, so $\triangle ABC \sim \triangle DEC$ by AA Similarity. Therefore, $AC/CD = BC/CE = AB/DE = 7/21 = 1/3$, so $AC = CD/3 = \boxed{16/3}$ and $BC = CE/3 = \boxed{4}$.

(b) Since $\triangle FGI \sim \triangle FHJ$ by AA (because $\overline{GI} \parallel \overline{HJ}$ gives us $\angle FGI = \angle FHJ$ and $\angle FIG = \angle FJH$), we have $GI/HJ = FG/FH = 4/13$. Therefore, $HJ = GI(13/4) = \boxed{91/4}$.

(c) Since $\angle LOM = 90^\circ - \angle NOM = \angle MNO$ and $\angle LMO = \angle OMN$, we have $\triangle LMO \sim \triangle OMN$ by AA Similarity. Therefore, $ON/LO = MN/OM = OM/LM = 1.2/1.6 = 3/4$, so $ON = OL(3/4) = \boxed{1.5}$ and $MN = OM(3/4) = \boxed{0.9}$.

(d) Since $\angle PQR = \angle PST$ and $\angle P$ equals itself, we have $\triangle PQR \sim \triangle PST$ by AA Similarity. So, $PQ/PS = PR/PT = 9/20$. Therefore, $PS = PQ(20/9) = 160/9$, so $RS = PS - PR = 160/9 - 9 = \boxed{79/9}$.

5.2.2 $\boxed{\text{Yes.}}$ Let the vertex angle of an isosceles triangle have measure x and each of the base angles have measure y. Then $x + 2y = 180^\circ$, or $y = 90^\circ - x/2$. Hence, if the vertex angles of each of two isosceles triangles have the same measure, x, then all four base angles of the two triangles have the same measure, $90^\circ - x/2$. Therefore, the triangles have the same angle measures and must be similar.

5.2.3

(a) Since $WXYZ$ is a square, we have $\angle WZY = \angle ZYX = 90^\circ$. Therefore, $\angle WZY + \angle ZYX = 180^\circ$, so $\overline{WZ} \parallel \overline{XY}$.

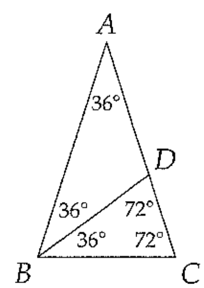
(b) Since $\overline{WZ} \parallel \overline{XY}$, we have $\angle AZM = \angle MYB$ and $\angle MAZ = \angle MBY$. Furthermore, we are given $ZM = MY$, so $\triangle AZM \cong \triangle BYM$ by AAS Congruence. Therefore, $AZ = BY$.

(c) From the triangle congruence in the previous part, we have $AM = BM$, so $\triangle AMX \cong \triangle BMX$ by SAS Congruence. Therefore, $XA = XB$.

(d) Since $WXYZ$ is a square, all of its angles are right angles. Specifically, $\angle Z = \angle XYM = 90^\circ$. Furthermore, since $\angle ZMY$ is a straight angle, we have $\angle AMZ + 90^\circ + \angle XMY = 180^\circ$, so $\angle AMZ = 90^\circ - \angle XMY$. From right triangle $\triangle XMY$, we have $\angle YXM = 180^\circ - \angle XYM - \angle XMY = 90^\circ - \angle XMY$ also, so $\angle YXM = \angle AMZ$. Therefore, $\triangle AZM \sim \triangle MYX$ by AA Similarity. Hence, $AZ/ZM = MY/XY$. Since $WXYZ$ is a square, we have $XY = ZY$, so $ZM = MY = XY/2$. Therefore, we have

$$AZ = \frac{(ZM)(MY)}{XY} = \frac{(XY/2)(XY/2)}{XY} = \frac{XY}{4}.$$

5.2.4



(a) Since $AB = AC$, $\angle ABC = \angle ACB$. From $\triangle ABC$ we have $\angle ABC + \angle ACB = 180^\circ - \angle BAC = 180^\circ - 36^\circ = 144^\circ$, so $\angle ABC = \angle ACB = 144^\circ/2 = 72^\circ$. Since $\angle ABD = \angle CBD$ and $\angle ABD + \angle CBD = \angle ABC = 72^\circ$, we have $\angle ABD = \angle CBD = 72^\circ/2 = 36^\circ$. Therefore, $\angle A = \angle DBC$ and $\angle ACB = \angle BCD$, so $\triangle ACB \sim \triangle BCD$ by AA Similarity.

(b) Let $x = AB$. Then $AC = x$. From $\triangle ACB \sim \triangle BCD$ we have $\angle BDC = \angle ABC$. We also have $\angle ABC = \angle ACB$ and $\angle ACB = \angle BCD$, so $\angle BDC = \angle BCD$. Therefore, $\triangle BCD$ is isosceles, and $BD = BC = 1$. Also, $\angle ABD = \angle BAD$, so $AD = BD = 1$. Therefore, $CD = AC - AD = x - 1$. The similar triangles from part (a) give us $AB/BC = BC/CD$, so we have

$$\frac{x}{1} = \frac{1}{x-1},$$

or $x(x-1) = 1$. Rearranging gives $x^2 - x - 1 = 0$. By the quadratic formula, we have

$$x = \frac{1 \pm \sqrt{5}}{2}.$$

But $\frac{1-\sqrt{5}}{2} < 0$ and x must be positive, so $AB = x = \boxed{(1+\sqrt{5})/2}$.

5.2.5 Since $\overline{AB} \parallel \overline{CD}$, we have $\angle A = \angle DCE$ and $\angle ABE = \angle CDE$, so $\triangle ABE \sim \triangle CDE$ by AA Similarity. Therefore,

$$\frac{4}{x} = \frac{5}{5 + AC}.$$

Cross-multiplying gives $20 + 4AC = 5x$, so $AC = \frac{5x}{4} - 5$. Similarly, $\overline{FG} \parallel \overline{AB}$, so $\triangle FGH \sim \triangle ABH$, which gives

$$\frac{7}{7 + y + 5 + AC} = \frac{5}{x}.$$

Substituting for AC gives:

$$\frac{7}{7+y+5x/4} = \frac{5}{x}.$$

Cross-multiplying gives $7x = 35 + 5y + 25x/4$, so $3x/4 = 35 + 5y$, or $x = \boxed{140/3 + 20y/3}$.

Exercises for Section 5.3

5.3.1 Since $AB/AD = AC/AE$ and $\angle BAC = \angle DAE$, we have $\triangle ABC \sim \triangle ADE$ by SAS Similarity. Therefore $BC/DE = AB/AD = 3/5$, so $DE = (BC)(5/3) = \boxed{10}$.

5.3.2 Since M is the midpoint of $\overline{FG}$, we have $FM = MG$. Since F is the midpoint of $\overline{JM}$, we have $FJ = FM$. Therefore, $JM = 2FM = 2MG$. Similarly, $IM = 2MH$. So, $IM/MH = JM/MG = 2$ and $\angle GMH = \angle JMI$, so $\triangle IMJ \sim \triangle HMG$ by SAS Similarity. Therefore $\angle IJM = \angle HGM$, so $\overline{IJ} \parallel \overline{GH}$.

5.3.3 Since $WZ^2 = (WX)(WY)$, we have $WZ/WX = WY/WZ$. Also, $\angle ZWX = \angle YWZ$, so we have $\triangle WZX \sim \triangle WYZ$. This similarity gives the desired $\angle WZX = \angle WYZ$.

5.3.4

(a) $\angle QBC = \angle QBP = 90^\circ - \angle QPB = 90^\circ - \angle RPC = \angle PCR = \angle QCB$.

(b) Because $\angle QBC = \angle QCB$, we have $QB = QC$. Since $QB = QC$ and $QA = QR$, we have $QB/QC = QA/QR$, so $\triangle QBC \sim \triangle QAR$ by SAS Similarity. Therefore, $\angle QCB = \angle QRA$ and $\overline{PB} \parallel \overline{RA}$ as desired.

Exercises for Section 5.4

5.4.1 We are given:

$$\frac{\text{Base length first triangle}}{\text{Leg length first triangle}} = \frac{\text{Base length second triangle}}{\text{Leg length second triangle}},$$

and a simple rearrangement turns this into

$$\frac{\text{Base length first triangle}}{\text{Base length second triangle}} = \frac{\text{Leg length first triangle}}{\text{Leg length second triangle}}.$$

Clearly, this equality holds with the other legs of the triangles, so the two triangles are similar by SSS Similarity. Therefore, the corresponding angles of the two triangles are the same. Specifically, the vertex angles of the two triangles are equal.

Exercises for Section 5.5

5.5.1 Since $\overline{XY} \parallel \overline{QR}$, we have $\angle PXY = \angle PQR$ and $\angle PYX = \angle PRQ$, so $\triangle PXY \sim \triangle PQR$ by AA Similarity. Therefore, $PY/PR = XY/QR = 1/3$, so $PY/(PY + 8) = 1/3$. Cross-multiplying gives $3PY = PY + 8$, so $PY = \boxed{4}$.

5.5.2

(a) The parallel lines in the diagram give us $\angle ECD = \angle FDB$ and $\angle EDB = \angle FBD$, so $\triangle FBD \sim \triangle EDC$. Since $[EDC] = 25[FBD]$ and the two triangles are similar, each side of $\triangle EDC$ is $\sqrt{25} = 5$ times each corresponding side of $\triangle FBD$. Therefore, $CD/DB = \boxed{5}$.

(b) Since $\overline{AB} \parallel \overline{ED}$, we have $\angle A = \angle CED$ and $\angle B = \angle EDC$, so $\triangle CAB \sim \triangle CED$. Since $CD/DB = 5$ from the first part, we have $CD/CB = CD/(CD + DB) = CD/(CD + CD/5) = 5/6$. Therefore, the ratio of corresponding sides in $\triangle CED$ and $\triangle CAB$ is $5/6$, and the ratio of their areas is $(5/6)^2 = \boxed{25/36}$.

(c) $\triangle EDC$ takes up $25/36$ of $\triangle ABC$. $[BDF] = [EDC]/25 = [ABC]/36$, so it takes up another $1/36$ of $\triangle ABC$, leaving $10/36 = 5/18$ of the area of $\triangle ABC$ for $AEDF$. The parallel lines give us $\angle EFD = \angle AEF$ and $\angle AFE = \angle FED$, which combined with $EF = EF$ gives $\triangle AEF \cong \triangle DFE$ by ASA. Therefore, $[AEF] = [AEDF]/2$. Since $[AEDF]$ is $5/18$ of $[ABC]$, we have $[AEF] = (5/36)[ABC]$, or $[AEF]/[ABC] = \boxed{5/36}$.

5.5.3

(a) Since $\overline{ZA} \parallel \overline{WX}$, we have $\angle AZC = \angle CXW$ and $\angle CWX = \angle CAZ$, so $\triangle CAZ \sim \triangle CWX$ by AA Similarity. Therefore, $ZC/XC = AC/CW$.

(b) Following essentially the same logic as in the previous part, we have $\triangle DXB \sim \triangle DZW$, so $XD/ZD = DB/WD$.

(c) Since $ZC = XD$, we also have $ZD = ZC + CD = XD + CD = XC$, so $ZC/XC = XD/ZD$. Combining this with both of the first two parts gives $DB/WD = XD/ZD = ZC/XC = AC/WC$. Adding 1 to both sides of $DB/WD = AC/WC$ gives $(DB + WD)/WD = (AC + WC)/WC$, or $BW/WD = AW/WC$. Therefore, we have $\triangle CWD \sim \triangle AWB$ by SAS Similarity. From this, we have $\angle WCD = \angle WAB$, so $\overline{CD} \parallel \overline{AB}$.

5.5.4

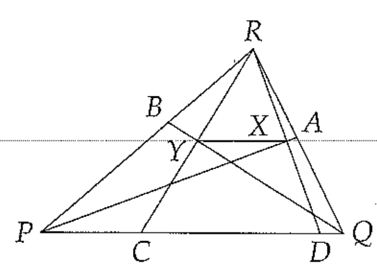
(a) Since $PR = PQ$, we have $\angle R = \angle PQR = \angle WQZ$. Since $\overline{ZX} \parallel \overline{QY}$, we have $\angle ZXR = \angle QYR = 90°$. So, $\angle ZXR = \angle ZWQ$, and we have $\triangle QWZ \sim \triangle RXZ$ by AA Similarity.

(b) First, we note that $RQ = RZ - QZ$, which looks a lot like the expression we want to prove. Since $\overline{XZ} \parallel \overline{YQ}$, we have $\angle RYQ = \angle RXZ$ and $\angle RQY = \angle RZX$, so $\triangle RYQ \sim \triangle RXZ$. This similarity gives us $RZ/ZX = RQ/YQ$, so $RQ = (RZ/ZX)(YQ)$. From $\triangle QWZ \sim \triangle RXZ$ in the last part, we have $RZ/ZX = QZ/ZW$, so $QZ = (RZ/ZX)(ZW)$. Substituting these into $RQ = RZ - QZ$ gives

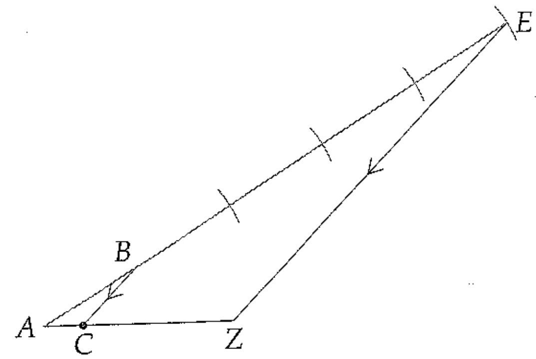
$$\frac{(RZ)(YQ)}{ZX} = RZ - \frac{(RZ)(ZW)}{ZX}.$$

Multiplying this equation by ZX/RZ gives the desired $YQ = ZX - ZW$.

5.5.5 We extend $\overrightarrow{RY}$ and $\overrightarrow{RX}$ to meet $\overline{PQ}$ at C and D, respectively, as shown. Since $\angle RPA = \angle APQ$, $PX = PX$, and $\angle RXP = \angle DXP$, we have $\triangle RXP \cong \triangle DXP$ by ASA Congruence. Similarly, we have $\triangle CYQ \cong \triangle RYQ$. Therefore, $RX = XD$ and $RY = YC$, so X and Y are midpoints of $\overline{RD}$ and $\overline{RC}$, respectively. So, we have $RY/RC = RX/RD = 1/2$, which gives us $\triangle RYX \sim \triangle RCD$ by SAS Similarity. Therefore, we have $\angle RYX = \angle RCD$, so $\overline{XY} \parallel \overline{DC}$. Since $\overline{DC}$ is on the same line as $\overline{PQ}$, we have $\overline{XY} \parallel \overline{PQ}$.

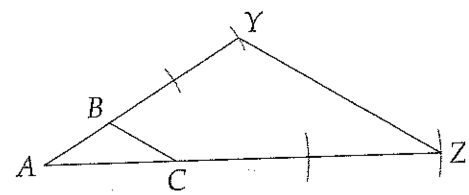


Exercises for Section 5.6



5.6.1 We follow essentially the same procedure as described in the text. Let our given segment of length 1 be $\overline{AZ}$. We then construct $\overline{AB}$ of any length (preferably small!), then copy it 4 times along $\overrightarrow{AB}$ as shown to get point E such that $AE = 5AB$. We then draw $\overline{EZ}$ and construct a line through B parallel to $\overline{EZ}$ (as described earlier in the text). Where this line meets $\overline{AZ}$ we call point C. Since $\overline{BC} \parallel \overline{EZ}$, we have $\triangle ABC \sim \triangle AEZ$. Since $AB/AE = 1/5$, we have $AC/AZ = 1/5$ from our similar triangles. Since $AZ = 1$, we have $AC = 1/5$, as desired.

To construct a segment of length $2\frac{2}{3}$, we start by again letting our segment with length 1 be $\overline{AZ}$. We copy it again along $\overrightarrow{AZ}$ to find point Y such that $AY = 2$. We then construct a segment with length 1/3 exactly as described in the text, then copy this length twice past point Y on $\overrightarrow{AY}$ to reach point X such that $AX = 2\frac{2}{3}$ as desired.



5.6.2 If $\triangle ABC \sim \triangle XYZ$ and $[XYZ] = 9[ABC]$, then the sides of $\triangle XYZ$ must be $\sqrt{9} = 3$ times as long as the sides of $\triangle ABC$. The easiest way to construct such a triangle is to start with $\triangle ABC$, then extend $\overline{AB}$ past B and $\overline{AC}$ past C. We locate point Y on $\overrightarrow{AB}$ such that $AY = 3AB$ by copying $\overline{AB}$ twice on $\overrightarrow{AB}$ past point B. Similarly, we find Z on $\overrightarrow{AC}$ such that $AC = 3AZ$. Since $AY/AB = 3$, $AZ/AC = 3$, and $\angle YAZ = \angle BAC$, we have $\triangle ABC \sim \triangle AYZ$ by SAS Similarity. The sides of $\triangle AYZ$ are three times as long as those of $\triangle ABC$, and the triangles are similar, so $\triangle AYZ$ has 9 times the area of $\triangle ABC$. Therefore, $\triangle AYZ$ is our desired triangle.

Review Problems

5.22

(a) Since $\overline{AB} \parallel \overline{CE}$, we have $\angle DAB = \angle DEC$ and $\angle DBA = \angle DCE$. Therefore, $\triangle DAB \sim \triangle DEC$ by AA Similarity.

(b) Since $\angle IHJ = \angle FHG$ (vertical angles) and $\angle GFH = \angle HIJ$, we have $\triangle HFG \sim \triangle HIJ$ by AA Similarity.

(c) There are no two triangles in this diagram that must be similar.

(d) Since $OP/OQ = OS/OR$ (both ratios equal 2/3) and $\angle QOP = \angle ROS$, we have $\triangle POQ \sim \triangle SOR$ by SAS Similarity.

(e) Since $\angle T = \angle W$ and $\angle U = \angle X$, we have $\triangle TUV \sim \triangle WXY$ by AA Similarity.

(f) Since $AB/FE = BC/DF = AC/DE$, we have $\triangle ABC \sim \triangle EFD$ by SSS Similarity.

5.23 Since $\angle A = \angle R$ and $\angle B = \angle P$, we have $\triangle ABC \sim \triangle RPQ$. Therefore, we have $AB/RP = BC/PQ = AC/RQ$. Substitution gives $y/12 = x/6 = 3/8$, from which we find $y = (12)(3/8) = \boxed{9/2}$ and $x = (6)(3/8) = \boxed{9/4}$.

5.24 Since $\overline{PQ} \parallel \overline{BC}$, we have $\angle APQ = \angle ABC$ and $\angle AQP = \angle ACB$. Therefore, $\triangle APQ \sim \triangle ABC$ by AA Similarity. Since $AB = 12$, $PB = 9$, and $AP = AB - PB$, we have $AP = 3$. Our similarity gives us $AQ/AC = AP/AB = 3/12 = 1/4$. So, $AQ = AC/4 = \boxed{9/2}$.

5.25 To maximize the perimeter of the second triangle, we should maximize the ratio of corresponding side lengths between the two triangles. If the side of length 4 cm in the first triangle corresponds to the side of length 36 cm in the second triangle, then this ratio is $36/4 = 9$. If it is the side of length 6 cm, then this ratio is $36/6 = 6$, and if it is the side of length 9 cm, then this ratio is $36/9 = 4$.

Hence, the maximum ratio is 9, and the maximum perimeter of the second triangle is $9 \cdot (4+6+9) = \boxed{171\text{ cm}}$.

5.26 Since $\overline{DB} \parallel \overline{EC}$, we have $\triangle DAB \sim \triangle EAC$. Therefore, we must have $AD/AE = AB/AC$. However, using the lengths given in the diagram, we have $AD/AE = 2/(6.5) = 4/13$ and $AB/AC = 3.5/9.5 = 7/19$. These two ratios are not equal! Therefore, the diagram given in the problem is impossible.

5.27 Since $AB/AE = 6/20 = 3/10$ and $AC/AD = 9/30 = 3/10$, we have $AB/AE = AC/AD$. This, combined with $\angle BAC = \angle EAD$ gives us $\triangle ABC \sim \triangle AED$. Therefore, $ED/BC = DA/AC$, so $DE/13 = 30/9$. Finally, we have $DE = 13(30/9) = \boxed{130/3}$.

5.28 Since $\angle PQR = \angle TSR$ and $\angle PRQ = \angle TRS$, we have $\triangle PQR \sim \triangle TSR$. Therefore, we have $PR/RT = QR/RS = PQ/ST$. We also have $PR = RS$ and are given a number of side lengths. Making these substitutions gives

$$\frac{PR}{12} = \frac{6}{PR} = \frac{8}{10}.$$

From $PR/12 = 8/10$, we have $PR = 9.6$. From $6/PR = 8/10$, we have $PR = (6)(10/8) = 60/8 = 7.5$. But PR can't have two different values! Therefore, the diagram in the problem is impossible.

5.29 From $\triangle WYZ$ we have $\angle YZW = 180^\circ - \angle WYZ - \angle YWZ = 90^\circ - \angle YWZ$. Also, $\angle XWY = \angle XWZ - \angle YWZ = 90^\circ - \angle YWZ$. Therefore, $\angle YZW = \angle XWY$. Together with $\angle ZYW = \angle WYX$, this gives us $\triangle WYX \sim \triangle ZYW$ by AA Similarity. Therefore, we have $WY/YZ = XY/WY$. Substitution gives $WY/6 = 4/WY$, so $WY^2 = 24$. Taking the square root of both sides gives $\boxed{WY = 2\sqrt{6}}$.

Similarly, $\angle YVZ = 90^\circ - \angle YZV = \angle WZY$ and $\angle VYZ = \angle WYZ$, so $\triangle VYZ \sim \triangle ZYW$ by AA. Therefore, $VY/YZ = YZ/WY$, so $VY = YZ^2/WY = (36)/(2\sqrt{6}) = 18/\sqrt{6} = \boxed{3\sqrt{6}}$.

5.30

(a) Since $BM = CN$ and $AB = AC$, we have $AB - BM = AC - CN$. Therefore, $AM = AN$.

(b) From our first part we have $AM = AN$. Since we also have $AB = AC$, we have $AM/AB = AN/AC$. This, combined with $\angle BAC = \angle MAN$, gives us $\triangle MAN \sim \triangle BAC$ by SAS Similarity. Therefore, the angles of $\triangle MAN$ equal those of $\triangle BAC$, so the angles of $\triangle MAN$ are each 60° as well. Thus, $\triangle MAN$ is equilateral. We also could note that since $AM = AN$ and $\angle A = 60^\circ$, we have $\angle ANM = \angle AMN = (180^\circ - \angle A)/2 = 60^\circ$.

5.31

(a) Since $\triangle ABC \sim \triangle YZX$ and the area of $\triangle YZX$ is 9 times the area of $\triangle ABC$, the side lengths of $\triangle YZX$ are $\sqrt{9} = 3$ times the corresponding side lengths of $\triangle ABC$. Therefore, $YZ = 3AB = \boxed{27}$, where the order of the vertices in the triangle similarity relationship tells us which side of $\triangle ABC$ corresponds to $\overline{YZ}$ in $\triangle YZX$.

(b) By the same reasoning as part (a), we have $XZ = 3BC = 36$. Letting our desired altitude have length h, we have $[YZX] = (h)(XZ)/2$. Since we are given $[YZX] = 360$, we have $(h)(XZ)/2 = 360$. Since $XZ = 36$, we have $h = \boxed{20}$. (We could also have found the corresponding height in $\triangle ABC$ and multiplied it by 3.)

5.32 Let $AE = x$, so $BE = 25 - x$. Since $\triangle AED \sim \triangle BCE$, we have $AE/AD = BC/BE$. Substitution gives $x/12 = 12/(25-x)$. Cross-multiplying gives $x(25-x) = 144$, so we have $x^2 - 25x + 144 = 0$. Therefore, we have $(x-9)(x-16) = 0$, so $x = 9$ or $x = 16$. When $x = 16$, we have $AE = 16$ and $BE = 25 - AE = 9$. We are given $AE < BE$, so we discard this possibility. For $x = 9$, we have $\boxed{AE = 9}$ and $BE = 16$.

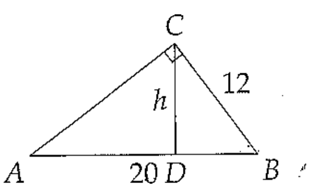
5.33 Since $\overline{WY} \parallel \overline{XR}$, we have $PY/YR = PW/WX = 3/2$. Since $\overline{XY} \parallel \overline{QR}$, we have $PX/XQ = PY/YR = 3/2$. Therefore, $XQ = (PX)(2/3) = \boxed{20/3}$.

5.34 Since $\overline{AP} \parallel \overline{RC}$, we have $\angle P = \angle CRB$ and $\angle PAB = \angle RCB$, so $\triangle BRC \sim \triangle BPA$. Therefore, $BC/BA = CR/PA$. Similarly, $\overline{BQ} \parallel \overline{CR}$ gives us $\triangle QBA \sim \triangle RCA$, so $AC/BA = CR/BQ$. Adding these two equations gives us

$$\frac{CR}{AP} + \frac{CR}{BQ} = \frac{BC}{BA} + \frac{AC}{BA} = \frac{BC + AC}{BA} = \frac{BA}{BA} = 1.$$

Dividing both sides of this by CR gives $\frac{1}{AP} + \frac{1}{BQ} = \frac{1}{CR}$, as desired.

5.35 Label the vertices A, B, and C, so that $BC = 12$, $AB = 20$, and the right angle is at C. Let D be the foot of the perpendicular from C to AB.



Then $\angle ACB = \angle ADC = \angle CDB = 90^\circ$. Also, $\angle ACD = 90^\circ - \angle CAD = \angle ABC$, and $\angle CBD = \angle ABC$. Therefore, triangles ABC, ACD, and CBD are similar.

Let $AD = x$, so $DB = 20 - x$. From $\triangle CDB \sim \triangle ACB$, we have $DB/BC = BC/AB$, so $20 - x = 12^2/20$, from which we find $x = 64/5$. Therefore, $AD = 64/5$ and $DB = 20 - x = 36/5$. From $\triangle ACD \sim \triangle CBD$, we have $CD/BD = AD/CD$, so $CD^2 = (AD)(DB)$. Therefore, $h = CD = \sqrt{(AD)(DB)} = \boxed{48/5}$.

5.36 Since $\overline{DE} \parallel \overline{BC}$, $\angle ADE = \angle ABC$ and $\angle AED = \angle ACB$. Then by AA similarity, triangles ABC and ADE are similar. Therefore,

$$\frac{AB}{AD} = \frac{AC}{AE}.$$

Since $AB = AD + DB$ and $AC = AE + EC$, we have

$$\frac{AD + DB}{AD} = \frac{AE + EC}{AE},$$

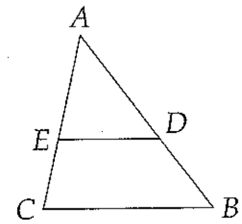
so

$$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}.$$

Subtracting 1 from each side gives $DB/AD = EC/AE$, and a little rearranging gives the desired $AD/AE = DB/EC$.

Challenge Problems

5.37 We have $AD/AE = BD/EC$. We would like to show that $AB/AC = AD/AE$, so we can then conclude that $\triangle ABC \sim \triangle ADE$ by SAS Similarity. A little algebra, and noting that we are given $AD = (AE)(BD)/EC$, does it for us:



$$\frac{AB}{AC} = \frac{AD + DB}{AE + EC} = \frac{\frac{(AE)(BD)}{EC} + BD}{AE + EC} = \frac{BD\left(\frac{AE}{EC} + 1\right)}{AE + EC}$$

$$= \frac{BD\left(\frac{AE+EC}{EC}\right)}{AE + EC} = \frac{(BD)(AE + EC)}{(EC)(AE + EC)} = \frac{BD}{EC}.$$

Since $BD/EC = AD/AE$, we have $AB/AC = BD/EC = AD/AE$. We also have $\angle BAC = \angle DAE$, so $\triangle ABC \sim \triangle ADE$ by SAS Similarity. Our similar triangles give us $\angle ABC = \angle ADE$, so $\overline{BC} \parallel \overline{DE}$.

5.38 The third angle (i.e. the other base angle) of each of the isosceles triangles equals 180° minus the sum of the other two angles. Therefore, the two 'other' base angles of each of the triangles are the same. Since both triangles have two base angles that have this measure, the two triangles have two angle measures in common. Therefore, the triangles are similar by AA Similarity.

5.39 The area of $CDEFG$ can be calculated by taking the area of triangle ABC and subtracting the areas of triangles ADE and BFG. To find the areas of our little right triangles, we need FG, FB, AE, and DE. We can use similar triangles to find FG. We have $\angle GFB = \angle CHB = 90^\circ$ and $\angle GBF = \angle CBH$, so triangles GFB and CHB are similar. Then

$$\frac{FG}{CH} = \frac{FB}{HB},$$

so

$$FG = \frac{CH \cdot FB}{HB} = \frac{24 \cdot 6}{18} = \boxed{8}.$$

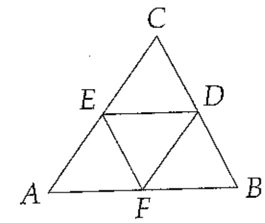
We have $DE = FG = 8$, and we know $FB = 6$. Since $AC = CB$, we have $\angle A = \angle B$. Together with $\angle AED = \angle BFG$ and $DE = GF$, this gives $\triangle DEA \cong \triangle GFB$ by AAS Congruence. Therefore, $AE = BF = 6$.

Hence,

$$\begin{aligned}[CDEFG] &= [ABC] - [ADE] - [BFG] \\ &= \frac{1}{2}AB \cdot CH - \frac{1}{2}AE \cdot DE - \frac{1}{2}BF \cdot GF \\ &= \frac{1}{2}(36 \cdot 24) - \frac{1}{2}(6 \cdot 8) - \frac{1}{2}(6 \cdot 8) = \boxed{384}.\end{aligned}$$

5.40 First, we rewrite our given equation as $AD/AE = AB/AC$. This, combined with $\angle DAE = \angle CAB$ gives us $\triangle DAE \sim \triangle BAC$ by SAS Similarity. Therefore, $\angle ADE = \angle ABC$, so $\angle CBE + \angle CDE = \angle CBA + \angle CDE = \angle ADE + \angle CDE = 180^\circ$, because $\angle ADE$ and $\angle CDE$ together make a straight angle.

We can also rewrite our given equation as $AD/AB = AE/AC$. This, combined with $\angle BAD = \angle CAE$ gives us $\triangle ABD \sim \triangle ACE$ by SAS Similarity. Therefore, we have $\angle ADB = \angle CEA$, so $\angle ADB + \angle BEC = \angle CEA + \angle BEC = 180^\circ$, as desired.



5.41 Because $\overline{FE} \parallel \overline{BC}$, we have $\angle DFE = \angle FDB$. Because $\overline{BF} \parallel \overline{DE}$, we have $\angle BFD = \angle FDE$. Combined with $DF = DF$, we have $\triangle BDF \cong \triangle EFD$ by ASA Congruence. Therefore, $BF = DE$. Similarly, we can show $\triangle AFE \cong \triangle DEF$, so $AF = DE$. Since $AF = DE$ and $BF = DE$, we have $AF = BF$, so F is the midpoint of $\overline{AB}$.

We could finish the problem with more pairs of congruent triangles, or we can use what we know about similarity. We'll show the latter approach. Since $\overline{FE} \parallel \overline{CD}$, we have $\triangle AFE \sim \triangle ABC$ by AA ($\angle AFE = \angle B$ and $\angle AEF = \angle C$), so $AE/AC = AF/AB = 1/2$. Therefore, E is the midpoint of $\overline{AC}$. Similarly, since $\overline{DE} \parallel \overline{AB}$, we have $\triangle CDE \sim \triangle CBA$, so $CD/CB = CE/AC = 1/2$, and D is the midpoint of $\overline{BC}$.

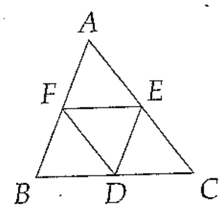
5.42 As described in the text, since $\overline{PS} \parallel \overline{QT}$, we have $QP/QR = TU/TR$. Similarly, since $\overline{PQ} \parallel \overline{ST}$, we have $SU/SP = TU/TR$. Combining these equations gives the desired $SU/SP = QP/QR$.

5.43 We are given that C is the midpoint of $\overline{YZ}$. Therefore, triangles XYC and XZC have equal bases $YC = CZ$ and the same altitude, so $[XYC] = [XZC]$. But $[XYC] + [XZC] = [XYZ] = 8$, so $[XYC] = [XZC] = 4$.

Next, let M be the intersection of $\overline{AB}$ and $\overline{CX}$. Since A is the midpoint of $\overline{XY}$, $XA = XY/2$. Since B is the midpoint of $\overline{XZ}$, $XB = XZ/2$. Therefore, triangles AXB and YXZ are similar. Then $\angle XAB = \angle XYZ$, so $\overline{AB}$ and $\overline{YZ}$ are parallel. Furthermore, $\angle AXM = \angle YXC$, so triangles XAM and XYC are similar. The ratio between their sides is $XA/XY = 1/2$, so the ratio between their areas is

$$\frac{[XAM]}{[XYC]} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Hence, $[XAM] = [XYC]/4 = 1$. Then the area of the shaded region is $[AMCY] = [XYC] - [XAM] = 4 - 1 = \boxed{3}$.



5.44 Let ABC be an arbitrary triangle, and let D, E, and F be the midpoints of $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Then $AF = AB/2$, $AE = AC/2$, and $\angle FAE = \angle BAC$, so by SAS, triangles FAE and BAC are similar with ratio 1/2. Hence, $FE = BC/2$.

Similarly, $DF = AC/2$ and $DE = AB/2$. Therefore,

$$DE + DF + EF = \frac{1}{2}(AB + AC + BC).$$

In other words, the perimeter of triangle DEF is half of the perimeter of triangle ABC.

In the original problem, let P_n denote the perimeter of the n^{th} triangle. Then $P_2 = P_1/2$, $P_3 = P_2/2$, $P_4 = P_3/2$, and so on, until $P_{10} = P_9/2$. We can combine these and write $P_n = \left(\frac{1}{2}\right)^{n-1} P_1$. Therefore,

$$\frac{P_{10}}{P_3} = \frac{(\frac{1}{2})^9 P_1}{(\frac{1}{2})^2 P_1} = \left(\frac{1}{2}\right)^7 = \boxed{\frac{1}{128}}.$$

5.45 Since $\overline{AB}$ and $\overline{CD}$ are parallel, $\angle EFG = \angle EAB$ and $\angle EGF = \angle EBA$. Therefore, triangles EFG and EAB are similar, so

$$\frac{AE}{EF} = \frac{AB}{FG} = \frac{5}{2}.$$