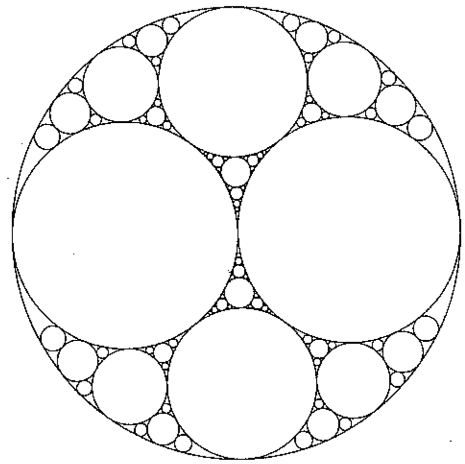

Hints to Selected Problems

1. Build a right triangle in which $\sin A = x$.
2. If a line is tangent to $\odot A$ at the point P, then the line is perpendicular to $\overline{AP}$.
3. Start with right triangle $\triangle TUV$ with right angle at U. Build a circle like the one we used in the previous parts.
4. To show that $AC = BC$ if $\overline{CM}$ bisects $\angle ACB$, try using the Angle Bisector Theorem.
5. Find the area of the whole figure two different ways.
6. What is $A'B'/AB$? What about $A'C'/AC$ and $B'C'/BC$?
7. The center of each circle must be on the graph of what equation?
8. Do you notice anything special about $\overline{AE}$ and $\overline{DF}$?
9. Consider the SSA examination we did in Problem 3.14.
10. Find the radii of the little circles. Draw segments from the center of the big circle to the points where the little circles are tangent to the big circle.
11. Let $\overrightarrow{VX}$ hit the base of the cone at Z. What kind of triangle is VXZ?
12. (For $3YZ > XY$.) Note that $\triangle XAB$ is equilateral. Apply the Triangle Inequality to $\triangle AYB$ and $\triangle YZB$.
13. How many little triangles do we add in the first step? In the second? In the third? In each step after the third? What is the ratio of the area of each triangle added to the area of each triangle added in the previous step?

14. Let the smallest exterior angle have measure x.

15. Why must $ACDF$ be a rectangle? (Don't forget to use $AD = 25$.)

16. Let the legs be a and b. And don't forget the expansion of $(a + b)^2$!

17. How did we find the sum of the exterior angles of a triangle? Can we try essentially the same tactic here?

18. Draw altitude $\overline{BX}$ of $\triangle ABC$. What kind of triangles are $\triangle ABX$ and $\triangle BXC$?

19. Draw the diagonal of the square that passes through the center of the little circle. Find the length of this diagonal in terms of the radius of the little circle.

20. Connect A, B, and C to the center of the circle of which the path is a part. Call this center O.

21. Let the triangle be $\triangle ABC$, and the point on the circumcircle be P. Let X and Y be the feet of the altitude from P to $\overline{AB}$ and $\overline{AC}$, respectively. Describe the circumcircles of $\triangle PXA$ and $\triangle PYA$. (Proving the existence of the Simson line is pretty tough! We'll explore it more in the next two volumes of this series.)

22. Build a right triangle with $\overline{XC}$ as one of the sides.

23. Let $\widehat{WZ} = x$. Label the other arcs in terms of x based on what we are given in the problem.

24. Prove that $\triangle BDF \cong \triangle EFD$. Can you show that any other triangles are congruent to $\triangle EFD$?

25. Notice that median $\overline{AM}$ is half as long as the side to which it is drawn.

26. If a translation maps E to C and F to B, then what must be true about $\overline{EC}$ and $\overline{FB}$?

27. Notice you have a midpoint of one side of $\triangle ABC$ – what do you know about midpoints and areas? What segment does this suggest drawing?

28. How are the sides of the pentagons related?

29. Try working backwards. Let our circles be $\odot O$ and $\odot P$ with Y on $\odot O$ and Z on $\odot P$ such that $\overline{YZ}$ is tangent to both circles. Furthermore, let r_O and r_P be the respective radii of the circles. Build right triangles.

30. Build a useful right triangle with the radius to A as hypotenuse.

31. Build more right angles.

32. Show that $EC = FB$ and $\overline{EC} \parallel \overline{FB}$.

33. Are our target angles corresponding angles of triangles we can prove are congruent?

34. Extend $\overline{AB}$ past B.

35. Extend $\overline{AQ}$ to point Z on $\overline{BC}$.

36. For the second part, notice that if we can find one set of a, b, and c for which it fails, then we are finished.

37. Suppose D is inside the circle. Extend $\overline{AD}$ and $\overline{CD}$ to meet the circle again at X and Y, respectively. In terms of arcs of the circumcircle, what is $\angle A + \angle C$? Can you prove this must be less than $180°$? What if D is outside the circle?

38. What do we get when we multiply the two areas together? What lengths are included in this product?

39. What is $[ABC]$?

40. Start with the Pythagorean Theorem, $a^2 + b^2 = 73^2$. Solve for a^2. Can you factor the result? Use clever trial and error to make your resulting expression a perfect square.

41. Let T be the center of the circle. Extend $\overrightarrow{VU}$ to hit the circle again and extend $\overleftrightarrow{UT}$ to hit the circle twice. Use Power of a Point.

42. Let A be the midpoint of $\overline{YZ}$, B be the midpoint of $\overline{XZ}$, and G be the centroid. Let $AG = x$ and $BG = y$. Find other lengths in terms of x and/or y.

43. What kind of triangles must $\triangle AOB$, $\triangle BOC$, $\triangle COD$, and $\triangle DOA$ be?

44. How did we prove that the area of a triangle equals its inradius times its semiperimeter? Can we do something similar here?

45. Find the ratio of the volume of each remaining piece to the volume of its original wedge.

46. Let $\angle C = x$ and $\angle D = y$. Label as many angles as you can in terms of x and y until you can write an equation.

47. Prove $AQ = RP$. Can you find some congruent triangles now?

48. Can you find $\angle BCA$ by considering what you know about $\triangle ABC$?

49. Draw one of the segments. How far are the endpoints of the segment from the center of the circle?

50. Can you find the volume of each piece we cut off?

51. If you were given several pieces that were shaped like the region described in the first hint, and you were given one piece shaped like the shaded region, could you build the square?

52. Deal with the hour hand and the minute hand separately. Where does the minute hand point? The hour hand?

53. Must all the interior diagonals pass through the same point? How far is this point from each vertex of the cube?

54. Write some equations involving $\angle RQZ$. For example, it can be combined with $\angle ZQP$ to make $\angle RQP$.

55. Build some right triangles.

56. What do you know about the little triangles on the outside?

57. Build a 30-60-90 triangle by dropping a well-chosen altitude.

58. Find similar triangles to express PX and YR in terms of sides whose lengths we know.

59. The centers of all the faces of the octahedron together are the vertices of what kind of polyhedron?

60. Consider how we built a right triangle for Problem 19.1.3.

61. Draw the altitude to the side of length 6. Can you find the length of this altitude?

62. Start with the power of point A and note that $AX = AW + WX$ and $AZ = AY + YZ$. Let $WX = YZ$ as given. Expand, rearrange, and factor, factor, factor.

63. Why must each plane of symmetry intersect one pair of faces in lines that are lines of symmetry of those faces?

64. What cross-section should you consider?

65. Solve for a and c in terms of b and d, then use the Pythagorean Theorem.

66. Unroll the rail!

67. This problem is the same as asking: 'I have a 7 by 7 square whose sides are blue. How can I cut it into 7 pieces of equal area such that each piece has the same total length of blue segments from the original square in its perimeter?'

68. Let the three polygons have a, b, and c sides, respectively. If we add the measure of an interior angle of each, what should we get? What should we get if we add the exterior angles?

69. What is $\angle STQ$?

70. Find more similar triangles. Draw $\overline{DB}$ and $\overline{EC}$.

71. Find and mark equal angles. See any congruent triangles?

72. Prove that the perimeter of the second triangle is half the perimeter of the first triangle.

73. We did a very similar problem in the text. Go back and study it for guidance.

74. In terms of s, the side length of the hexagon, what are the areas of the regions inside the hexagon but outside $ABCE$?

75. No matter where C is, $\overline{AB}$ is always the same. Given that $AB = 2$, what must we determine to find the area of $\triangle ABC$?

76. What kind of triangle is $\triangle GJM$?

77. What is $[XYC]/[XYZ]$?

78. To find the area of $\triangle AOB$, draw altitude $\overline{OX}$ from O to $\overline{AB}$. What kind of triangle is $\triangle BOX$?

79. $\overline{OB}$ and $\overline{OA}$ are radii of the same circle.

80. Let P and Q be the circumcenters of triangles ABE and BCE. What is the relationship between $\overline{PQ}$ and $\overline{BD}$?

81. Draw a picture. To do so, figure out the distance from the center of the man's circle to the closest point of grass beneath the man's hat, then figure out the distance from this center to the farthest point of grass beneath the man's hat.

82. What does $BC = AC = DC$ tell us?

83. Look at how you solved Problem 12.1. Try drawing the same extra line here.

84. (For $3YZ > XY$.) Draw $\triangle XYA$ and $\triangle XZB$ outside $\triangle XYZ$ such that $\triangle XAY \cong \triangle XYZ \cong \triangle XZB$. What kind of triangle is $\triangle XAB$?

85. How are $\angle WYX$ and $\angle WXY$ related? How are $\angle WYX$ and $\angle ZYX$ related?

86. Is $\triangle ADF$ equilateral?

87. What is $\angle ABP$?

88. Do you see any triangles that look congruent?

89. What is $[BXY]/[BXA]$?

90. Connect O to the midpoints of $\overline{AB}$ and $\overline{CD}$.

91. Write the Power of a Point relationship in terms of ratios.

92. Be careful; this is a little different than the similar problem in the text. Specifically, the fold connects points on opposite sides of the rectangle!

93. Can you consider the desired length as the altitude of a tetrahedron?

94. Look at the diagrams for constructions in this section. See any 90° angles?

95. Length ratios and medians should make us think of using centroids. Draw median $\overline{CK}$.

96. Why is $\overline{MN} \parallel \overline{BC}$?

97. Consider triangles $\triangle ACO$ and $\triangle BDO$.

98. Write $[AXC]$ and $[BXC]$ in terms of the areas you used for your area ratios in the previous hint. Use some clever algebra and the ratio statements you came up with in the last hint to show that $[AXC]/[BXC] = AF/FB$.

99. Draw the altitude from E to $\overline{AB}$. See any similar triangles?

100. To show all the diagonals pass through the same point, consider the midpoint, O, of one of the interior diagonals. Show that this point is mid-way between each pair of opposite faces of the cube.

101. Start with the Pythagorean Theorem, $a^2 + b^2 = 97^2$. Solve for a^2. Can you factor the result? Can you find a value of b that makes both factors perfect squares?

102. Call the length of one side of the original poster x. What are the areas of the old poster and the new poster in terms of x?

103. Use Power of a Point to show that $WX = YZ$ and $BX = CZ$.

104. Consider Problem 13.22. Find the power of the point with respect to each circle. Can they possibly be the same?

105. When you draw BCD and its image $B'C'D'$, you should have 6 outer triangles and a hexagon in the middle. What do you know about those 6 outer triangles?

106. Problems involving regular hexagons can often be made easier by dissecting the hexagon into 6 equilateral triangles.

107. Let $\overline{AF}$ and $\overline{EG}$ meet at X. Look at the angles of $ACEX$.

108. What do we know about triangles $\triangle WXY$ and $\triangle YZW$ that might be useful?

109. Extend $\overline{AB}$ past B; call the point where this extension meets m point H.

110. Consider the areas of triangles PBC, PCA, and PAB.

111. Unfold the tetrahedron.

112. We want to show that $(AB + CD)/MN = 2$. Find ratios equal to AB/MN and CD/MN.

113. The order of the vertices in the statement $\triangle ABC \sim \triangle ADB$ is important! Use them to write an equation in terms of side lengths.

114. Connect the vertices of the hexagon to the center of the circle. What kind of triangles do you form?

115. Under reflection through a plane of symmetry, what is the image of the vertex of the cone? The base of the cone? The center of the base of the cone?

116. What kind of triangle is $\triangle ABC$?

117. Take another look at Problem 8.31.

118. What do you know about the diagonals of a kite?

119. Let our legs have lengths x and y. Write two equations for x and y.

120. Let the regular polygon be $A_1A_2A_3 \cdots A_n$ and let O be the point where the angle bisectors of $\angle A_nA_1A_2$ and $\angle A_1A_2A_3$ meet. Can you prove $OA_1 = OA_2$? Can you use this to prove $OA_2 = OA_3$?

121. Let $\widehat{AC}$ and $\widehat{BD}$ meet at Y, and let $\widehat{BD}$ meet $\overline{AC}$ at X. Start with sector ABC and take out pieces you know how to handle.

122. Consider Problem 13.22.

123. What is the second angle? The third? The nth? What is the sum of these measures? What must this sum equal?

124. What is the 'blue' perimeter of each piece? Area of each piece?

125. $\overline{PD}$ is an angle bisector.

126. This problem is not nearly as hard as it looks. Don't try to find the nonoverlapping areas; try to find their difference.

127. In terms of arcs of the incircle, what is $\widehat{EDF}$? How about $\angle A$?

128. Show that MQ and MR both equal MT.

129. Build a 45-45-90 triangle by dropping a well-chosen altitude.

130. If we count all the sides of the pentagons, and all the sides of the hexagons, how many times do we count each seam?

131. Draw the altitudes from the endpoints of the shorter base.

132. Draw a line through B and the vertex of the square from the first hint that is not on a side of $\triangle ABC$. Let this line hit $\overline{AC}$ at F. Can you build the desired square with F as a vertex? (And more importantly, can you prove this is your desired square?)

133. Find EG and FH.

134. Find both $\angle YCZ + \angle YBZ$ and $\angle YAZ$ in terms of angles at Y and Z. (For example, consider $\triangle YCZ$.)

135. Let the smallest angle be x. What are the other two angles in terms of x?

136. Find $\angle ACD$ and $\angle BCF$ in terms of $\angle A$ and/or $\angle B$. (Don't forget what you know about the median to the hypotenuse of a right triangle!)

137. What Pythagorean triples have 50 as the hypotenuse? (We also strongly suggest trying to find an algebraic solution! Let x be the initial height of the top of the ladder and y the initial distance from the wall to the base.)

138. Forget about everything after C_1 and just find CC_1 and AC_1. Next, just look at right triangle CC_1A with altitude $\overline{C_1C_2}$ and just find C_1C_2. Then take another step and find C_2C_3.

139. Consider a cross-section that contains the common axis of the cones.

140. What kind of triangle is $\triangle ABC$?

141. Each segment in the first diagram is broken into how many segments in the second diagram? What is the ratio of the length of each segment in the first diagram to the length of each segment in the second?

142. Review the diagrams that showed us that SSA failed. See if you can play with the diagrams to find the cases where SSA succeeds. There's one tricky case that's not suggested in these diagrams. In what case could we determine another pair of corresponding angles are equal?

143. You have two cases to consider. Can the hypotenuse be odd if both legs are even? Can we have an even hypotenuse if one leg is even and the other is odd? (Use the Pythagorean Theorem!)

144. Let $ABCD$ and $EFGH$ be opposite faces of the cube, with $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$ as edges of the cube. Let N be the midpoint of $\overline{EG}$. Show that $ON = AE/2$ and that $\overline{ON}$ is perpendicular to face $EFGH$.

145. Don't forget that $\overline{OA}$ and $\overline{CD}$ are parallel!

146. Let $\overline{AC}$ meet $\overline{BD}$ at E. Find AE/EN. Find EN/NC.

147. Can you find some similar triangles?

148. Don't forget there are three different angles that could be the vertex angle!

149. Reinterpret the problem in terms of area and perimeter.

150. Let $AB = x$. What is BD? What is AD? What is CD in terms of x?

151. Let the triangle be $\triangle ABC$, with $\angle A$ as the largest angle. Write an equation with the information given.

152. Extend $\overrightarrow{RY}$ and $\overrightarrow{RX}$ to meet $\overline{PQ}$ at C and D, respectively.

153. The parallel lines of the previous part give us similar triangles.

154. Is it possible for a plane of symmetry to pass through exactly 1 vertex of the tetrahedron? How about 3? 4? 0?

155. Is there another tetrahedron with the same volume?

156. Use your equations to write $\angle ZQR$ in terms of $\angle PQR - \angle PRQ$.

157. Try adding all your equations together.

158. Are $\overline{XN}$ and $\overline{YN}$ corresponding sides of congruent triangles? Which triangles? Why are they congruent?

159. Draw a square with opposite vertices B and O.

160. Note that $(a+b)^2 = a^2 + 2ab + b^2$.

161. What is the length of the altitude to the side of length 10 cm of the first triangle?

162. Let the point where $\overline{XC}$ and $\overline{AB}$ meet be M. What is $[XAM]/[XYC]$?

163. From the last hint, or from the diagram in Problem 3.14, you might have deduced that BC_1C_2 is isosceles. Show that $\angle BC_1A + \angle BC_2A = 180^\circ$, then consider the sum of the angles in your two possible triangles.

164. Let the cube have edge length s. What is the length of an edge of the octahedron?

165. Draw $\overline{BD}$. Do you have a pair of congruent triangles?

166. Use your equation to find DE/AB. Don't forget that $AB = CD = CE + DE$!

167. Are triangles XAY and YBX congruent?

168. Show that $\triangle AA'A'' \cong \triangle XX'X''$.

169. In the previous part, you should have shown that if X is on $\overline{BC}$, then $AB^2 \le AC^2 + BC^2$, which is the opposite of what we are given. Follow similar steps to show the same is true if X is beyond B on $\overrightarrow{CB}$. What does this tell us?

170. Find two pairs of similar triangles. You may need to use a little algebra!

171. There are lots of parallel lines. How are the two triangles in this part related?

172. Note that $RQ = RZ - QZ$, which looks a lot like what we want.

173. What is $AX + BY$?

174. The region described in the first hint is part of sector AOD.

175. Consider the cross-section that also includes the points where the spheres touch the wall and the floor.

176. Show that $DQ = AP = PB = CR$.

177. Let O be the center of the scoop, A be the vertex of the cone, and B be a point on the circumference of the cone. What kind of triangle is $\triangle OBA$? What is $\angle OAB$?

178. Can you prove that $VY = VZ$?

179. Show that $AD/AE = AB/AC$. Note that $AB = AD + BD$ and $AC = AE + EC$, and that you are given an equation you can use to get an expression for AD.

180. Consider the reflection of $\angle AOC$ over $\overline{XY}$.

181. Approach 1: Is there a single line through which all three cuts pass?

182. Look back at the ways to prove two lines are parallel. Then consider $\angle Z$ and $\angle XYZ$.

183. M is the midpoint of $\overline{BD}$ if $BM = MD$. Are these segments corresponding parts of congruent triangles?

184. In the Review Problems of this chapter, we found the radius of a sphere inscribed in a regular tetrahedron. Will the same approach work here?

185. Consider a cross-section that contains the apex of the original pyramid, the center of the base, and the midpoints of opposite sides of the base.

186. Let the dimensions of the prism be a, b, and c. Write equations for the given information, then find abc and use that to find each of the dimensions.

187. Consider a circle centered at M with radius 8.

188. Find FG first, then find RG.

189. Use the first part to find NM/BM. Compare this ratio to AM/AB. Do you have a pair of similar triangles?

190. How is $[FOB]$ related to $[DOG]$?

191. Draw $\overline{BE}$.

192. Let each of the marked angles have measure x. Find $\angle BAE$ in terms of x.

193. What do you know about the diagonals of a kite?

194. Continue $\overline{YN}$ to hit $\overline{XZ}$ at R. What do we know about $\overline{YR}$?

195. Draw a radius of the small circle to a point of tangency, and a radius of the large circle to a vertex of the hexagon. What kind of triangle can you thus form?

196. What do you know about the third angle of each triangle?

197. Do any triangles in the diagram look congruent? (You may have to consider points that aren't labeled!)

198. Show that $\triangle GEH \sim \triangle AED$, where E is the midpoint of $\overline{BC}$.

199. Draw a circle with center C and a judiciously chosen radius.

200. What are the exterior angles in terms of $\angle A$, $\angle B$, and $\angle C$?

201. Can you construct a square that has one side on $\overline{BC}$ and one vertex on $\overline{AB}$? Try using this square to find the square you want.

202. Chase both angles. Relate $\angle QCB$ and $\angle QBP$ to other angles in the diagram.

203. Draw a line segment and place the points on it. Label all the distances you can determine.

204. Let O be the intersection of the perpendicular bisectors of $\overline{HI}$ and $\overline{GH}$. Show that $\triangle OIJ \cong \triangle OGH$.

205. Let W, X, Y, and Z be the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{CD}$, and $\overline{BD}$, respectively. What kind of quadrilateral is $WXYZ$? What is WX?

206. What is $\angle OMP$?

207. What kind of triangle is $\triangle EGH$?

208. Focus on the base BCD. What happens when this is rotated 60° about its center?

209. What is $\angle X + \angle Y + \angle Z$? How are $\angle X$ and $\angle Y$ related?

210. Let $\overline{XY}$ meet $\overline{AB}$ at M. Show that $\triangle AMX \cong \triangle AMY$.

211. Each resulting piece has five faces. Carefully draw a diagram and use it to figure out what sort of shapes these faces are. (Don't forget what shape each face of the regular tetrahedron is!)

212. What is $[AMB]$?

213. What is $([ADF] + [BFC])/[ABCD]$?

214. Can you prove that Y is the midpoint of $\overline{FH}$?

215. Build a 45-45-90 triangle by dropping a well-chosen altitude. You may have to extend a side.

216. For the 'only if' part, draw altitudes from the short base to the long base and find congruent right triangles.

217. Let X be the leftmost point of the shaded region. Can you find the area of the region bound by $\widehat{AX}$, $\widehat{BX}$, and $\overline{AB}$?

218. Just unrolling the outside of the cylinder isn't enough; Arnav has to go inside the glass!

219. Look back at how we have done problems in this chapter with frustums.

220. Show that $\triangle XYL \cong \triangle XZL$. (Why must $\widehat{YL} = \widehat{ZL}$?)

221. Let O be the center of the largest semicircle. What are the following in terms of r, s, and t: the radius of the largest semicircle, OX, OB, and OY? What kind of triangle is $\triangle OXY$?

222. What is $\angle PTQ$?

223. Find $[PXQ]/[ABC]$ in terms of PQ and BC.

224. Did you see a similar problem in the text?

225. Let there be k right angles. Note that each of the other angles must be less than 180°. So, the sum of the angles must be less than what quantity?

226. Can you find QD?

227. Can you find the lengths of the sides of the triangle? What kind of triangle is it?

228. Since $\odot O$ is tangent to $\overline{AB}$ and $\overline{AC}$, what do we know about the segment connecting A to the center of the circle?

229. What kind of triangle is $\triangle ABC$?

230. The first hint was for the slick method. The rest of the hints are for a more mechanical route. How far is the center of the sphere from a face of the tetrahedron? (Note: This is the same as asking what the radius of the sphere inscribed in the tetrahedron is.)

231. What two area ratios do we know are equal to AF/FB?

232. Combine your expressions for $\angle ACE$ and $\angle BCD$ to find $\angle DCE$.

233. Still don't have it? Time for a different approach. Let O be the center of each square. Connect O to each vertex of $ABCDEFGH$. Can you find the areas of the triangles you form?

234. When the ball bounces off the rail, how far will the center be from the rail?

235. What kind of triangle is $\triangle AGB$? Can you find an expression for BG? How about CG and DG?

236. Draw the figure. Pay close attention to the vertices of the second square.

237. Draw $\overline{OD}$. What kind of triangle is $\triangle AOD$?

238. Your desired region is two cones with the same base. What segment in your cross-section corresponds to the radius of this common base of the two cones? What does your diagram for finding this radius have in common with Problem 5.17?

239. Consider the circle with diameter $\overline{OP}$. Through what other points in the diagram must this circle pass?

240. What general type of quadrilateral might be useful?

241. Is there an angle in the diagram equal to $\angle AOC$ that is easy to find?

242. Let ABC and ADE be faces of the octahedron with centers G and H, respectively. Let M and N be the midpoints of $\overline{BC}$ and $\overline{DE}$. Find the following: MN, AG/AM, GH.

243. Prove $\triangle SPD \cong \triangle QPA \cong \triangle SRC \cong \triangle QRB$.

244. What is $[WPX]/[WXYZ]$?

245. Let one polygon have a sides and the other have b sides. Write two equations using the given information.

246. Consider quadrilateral $OIHG$.

247. We have lots of right angles, and we're looking for an angle. Maybe the Pythagorean Theorem will help. Build right triangles.

248. Look back at Problem 2.14.

249. What type of cevian is $\overline{BE}$ in $\triangle ABC$?

250. What kind of quadrilateral is $ABCD$?

251. Let $[SPC] = x$ and $[CPR] = y$. What is $[SPC]/[BPC]$? Use this to build an equation.

252. $DF = DE + EF$.

253. Approach 2: How do our regions of cheese after the cuts correspond to orderings of x, y, and z?

254. Prove that $\triangle RYX \sim \triangle RCD$. (You may need to find some congruent triangles first!)

255. What do you get if you add this ratio to the ratio in the previous part?

256. Find the angle between $\overline{AA_1}$ and $\overline{B_1C_1}$ in terms of arcs of the circle. Don't forget that $\widehat{B_1A} = \widehat{B_1C}$, $\widehat{AC_1} = \widehat{BC_1}$, etc.

257. Find EH/EF using similar triangles.

258. Find two pairs of similar triangles. Write two equations involving x, y, and AC.

259. Don't just hunt blindly for solutions once you have an equation set up – some of the solutions are very surprising. Find an organized way. Here's a hint: If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, and x, y, and z are positive integers, then at least one of these integers must be 3 or smaller (otherwise, the sum $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ will definitely be smaller than 1).

260. Find a pair of similar triangles such that the segments in $BE^2 = (AE)(DE)$ are among the sides of the triangles.

261. What is $[XPQ]/[XBC]$ in terms of PQ and BC?

262. Forget about A for a minute and pretend we know where B is. What is the shortest path that goes from B to a point on $y = 6$, then on to point D?

263. Similar triangles will help you find the side length of the second square base of the frustum.

264. Let $\overline{CE}$ meet $\overline{OA}$ at Y. What type of triangle is $\triangle OCY$?

265. Can you find the area of $\triangle MEN$?

266. Let $AB \leq AC$. (The proof for $AC < AB$ is essentially the same.) Arrange the triangles so that A and A' coincide and B and B' coincide. Does $\overline{AC'}$ intersect $\overline{BC}$? Why or why not?

267. Draw $\overline{OX}$ and $\overline{PY}$. Draw a line through P parallel to $\overline{XY}$. Consider the right triangle you thus form.

268. Consider the triangle formed by connecting the center of the sphere, the center of face ABC, and the midpoint of $\overline{AB}$. The hypotenuse of this triangle is the desired radius.

269. Find some similar triangles. You may want to rearrange the equation you're given.

270. Let $\odot O$ be the circle tangent to the four given circles. Apply the Triangle Inequality to $\triangle OPR$.

271. Let X be the point where the wall meets the floor and let M be the midpoint of the ladder. How is MX related to the length of the ladder?

272. The region described in the last hint is part of sector ABC.

273. Focus on a single side of the first triangle. What happens to it as we go from the first figure to the second figure?

274. Can you find three congruent triangles that have $\overline{DH}$, $\overline{HF}$, and $\overline{DF}$ as corresponding sides?

275. Draw $\overline{MC}$. What is $[MUC]/[NUC]$?

276. Let one of the legs have length x. Write an equation.

277. Extend the sides of the octagon; what type of quadrilateral is formed?

278. Make sure you track the path of the center of the ball, not the edge of the ball. You shouldn't be aiming at the reflection of B over the rail, because the ball will bounce before the center reaches the rail!

279. The points of tangency are the vertices of what figure?

280. Show that $\triangle ADE$, $\triangle EDF$, and $\triangle FDC$ are congruent.

281. What are two ways you can find the volume of tetrahedron $ABCF$?

282. For the hour hand, can you figure out how much of the distance from 11 to 12 the hour hand has covered by considering how much of the 11 o'clock hour has transpired?

283. Connect M to the midpoint of $\overline{O_1O_2}$.

284. You have to use the fact that the little circle is tangent to the largest semicircle somehow. What does this fact tell us? What segment should we draw to use this fact?

285. What is $[ABX]/[ABC]$?

286. Draw $\overline{CP}$.

287. Let M be the midpoint of $\overline{BC}$ and N be the midpoint of $\overline{CD}$. How is $\triangle AOP$ related to $\triangle AMN$?

288. Let $\angle ABO = x$, $\angle OBC = y$, and $\angle OAC = z$. Find other angles that equal these. Find $x + y + z$.

289. What arcs must you show are equal in order to show that $\angle XAM = \angle YAM$?

290. What kind of triangle is $\triangle MOY$? $\triangle NOX$?

291. Connect the endpoints of the chords to the center of the circle. What kind of triangles do you form? What is $ABCD$?

292. View the frustum as the result of chopping a small cone with radius r_1 off the top of a cone with radius r_2.

293. To show that $\overline{XL}$ is a diameter of the circle, note that $XZLY$ is a cyclic quadrilateral. What does this tell us about $\angle XZL + \angle XYL$?

294. Let the radius of the circle be r. Find PB in terms of r. How can you use Power of a Point?

295. Let X be the center of the small circle on the right, and let Y be the foot of the perpendicular from X to $\overline{AC}$. Let r be the radius of the little circle, and let s and t be the radii of the smaller semicircles. Find XY in terms of r, s, and t in two different ways.

296. What triangle is similar to $\triangle EMC$?

297. Break the problem into cases in terms of the types of cross-sections formed by the intersecting plane.

298. Prove that both AP and RQ equal $YZ/2$.

299. See Problem 8.4.3.

300. Prove that $DE/EF = DF/DB = DF/DE$.

301. If you answered the first part using trial and error, go back and read those hints!

302. Let point X be some point besides B on line m. Consider $\triangle OBX$ and show that $OX > OB$. Does this prove that m can't hit the circle a second time?

303. Lots of right angles and parallel lines mean lots of similar triangles. Use similar triangles to find as many lengths as you can.

304. To find the area of $\triangle AOX$, draw an altitude from A. What is $\angle AOX$?

305. Let O be the circumcenter of $\triangle JKL$. Why must point O be on the altitude from J to $\overline{KL}$? Let $OK = x$. Find some other lengths in terms of x.

306. You're looking for a ratio, so find some useful similar triangles.

307. How is CC_1 related to BC? How is C_1C_2 related to CC_1? And so on.

308. Let the midpoint of the last hint be N. What does MN equal?

309. $[PAEC] = [PAE] + [PEC]$.

310. Let V be the vertex of the cone, O be the center of the base, and X a point on the circumference. What kind of triangle is $\triangle VOX$? What is VX?

311. There's nothing special about A!

312. Draw an altitude from Q to $\overline{PR}$.

313. Show that $\angle PAC = \angle PCA = \theta/4$.

314. Are the triangles similar?

315. Find some congruent triangles with $\overline{YX}$ and $\overline{WZ}$ as corresponding sides.

316. Draw a segment representing one of the longest lines-of-sight. Build a right triangle by connecting the common center of the semicircles to the point of tangency of the segment to the smaller semicircle, and connecting the center to an endpoint of the segment.

317. Compare the sides of $\triangle ACP$ to those of a 30-60-90 triangle. What similarity theorem can you use?

318. Find the edge lengths of the area of intersection. What sort of shape is the intersection space?

319. What is $[AHF]/[AEF]$?

320. Consider a 60° rotation about R.

321. What's left over when you take $\triangle EDC$ and $\triangle FBD$ away from $\triangle ABC$? What portion of this is $\triangle AEF$?

322. The triangle is isosceles, so the perpendicular bisector of $\overline{AC}$ is also an altitude, median, and angle bisector. Can you find the length of the altitude from B to $\overline{AC}$?

323. Extend $\overrightarrow{PO}$ to hit the circle again at Y.

324. Write the given area equation in terms of sides (or parts of sides) of the rectangle.

325. Let $\overline{ZX}$ meet $\overline{AY}$ at B and $\overline{ZD}$ meet $\overline{AY}$ at C. Show that $\triangle ZBC \cong \triangle ZYC$.

326. Can you dissect the original triangle and rearrange it to make another triangle with two sides of length 13?

327. Look at how we did a very similar problem in the chapter, then try to find the distance from the corner of the room to the center of the sphere in two different ways.

328. What kind of quadrilateral is $WXYZ$?

329. Let the smallest square have side length x. Find all of the lengths in the diagram in terms of x.

330. Let P be the center of the semicircle with $\overline{BC}$ as diameter. What kind of triangle is $\triangle XYP$?

331. Start with the circumcircle of $\triangle ABC$.

332. What kind of triangle is $\triangle ACF$?

333. Show that both M and N are on the median of the trapezoid.

334. What sort of shape do we form if we connect the centers of the little spheres?

335. What other portions of the diagram have area equal to the shaded area?

336. Write each of the arcs in terms of $\overparen{PQ}$. Can you find the measures of the arcs now?

337. At what point on each face is the sphere tangent to the octahedron?

338. Pretend you already know what point on $\overrightarrow{BC}$ to go to. What point should you aim for to find the point on $\overrightarrow{BD}$ to go to? Once you know that, how can you figure out what point on $\overrightarrow{BC}$ to go to?

339. Parallel lines mean similar triangles. Let the diagonals meet at point E.

340. Let the radii of the small semicircles be s and t. Draw the full circle with diameter $\overline{AC}$. Continue $\overline{BD}$ to meet this circle again at E. What do we know about BE and BD? What tool considering lengths and circles can we use to find BD in terms of s and t?

341. Show that $ABCD$ and $A'BCD'$ are congruent trapezoids by showing that all the corresponding angles and sides of the two are the same.

342. What kind of triangle is $\triangle XQZ$?

343. Show that $PQ = PR$ and $PX = PY$. Use these to find another pair of equal lengths.

344. Squares of side lengths and a rectangle suggest building right triangles. Lots of them.

345. After substituting your expression for AB into your expression for AC^2/MO^2, you should be able to factor out BC^2 in the numerator. Try multiplying both the numerator and the denominator of the resulting expression by NO^2. You should then have an $NO^2 - MN^2$ term in both the numerator and denominator. Cancel them!

346. Triangles $\triangle AEB$ and $\triangle PEA$ share an altitude from E.

347. Let the ladder be $\overline{AB}$. What kind of triangle is $\triangle ABX$? How is $\overline{XM}$ related to it?

348. Let I be the incenter of $\triangle ABC$ and X be the point where the incircle is tangent to $\overline{AC}$. What are the lengths of the sides of $IXAZ$?

349. Use similar triangles to show that $CQ/PQ = AC/PB$.

350. Find similar triangles that have $\overline{CD}$ and $\overline{BD}$ as corresponding sides.

351. Let $\odot O$ be the circle that is tangent to our given three circles. Either $\angle OQR$ or $\angle OQP$ is at least 90°.

352. You should already have YZ. Let the distance from X to the point where the incircle touches $\overline{XZ}$ be x and don't forget that $\triangle XYZ$ is a right triangle.

353. Try using area.

354. Let the radius of the sphere be r. What is the radius of the cylinder? The height of the cylinder?

355. We know a lot about triangles. Consider building a triangle by extending one of the segments.

356. What is the angle between where the minute hand points and a line from the center of the clock to the '12' on the clock? How about the hour hand?

357. Let C_1 and C_2 be the two potential point Cs. What kind of triangle is BC_1C_2? What does this tell us about $\angle BC_1A$ and $\angle BC_2A$?

358. We know a whole lot about the angles of a triangle now. Build some triangles.

359. Rotate the heptagon so that the image of D is on $\overrightarrow{AC}$.

360. What angles equal $\angle ZQW$? What angles equal $\angle R$?

361. Find the ratios of $[PBS]$, $[PQA]$, and $[ARB]$ to $[PQRS]$. How is the sum of these ratios related to $[ABP]/[PQRS]$?

362. Let the side length of the hexagon be s. Find the ratio of the area of a square to the area of the hexagon. How about the area of one of the triangles? (Try doing these without actually finding s!)

363. What triangle similarity would allow you to deduce $\overline{DE} \parallel \overline{BC}$? What ratio of sides must you show is equal to AD/AE to deduce these triangles are similar?

364. After using Power of a Point, write everything in terms of PQ, PX, and RX. Rearrange and do some clever factoring.

365. Find the squares of the side lengths of $\triangle ACF$ in terms of edges of the prism.

366. Proving that $\overrightarrow{EY}$ will pass through G is the same as proving that $\overline{EG}$ passes through Y.

367. What is the ratio of the area of the large equilateral triangle to the area of one of the small equilateral triangles?

368. Must the diagonal connecting the incenter of $\triangle WXA$ to the incenter of $\triangle YZA$ pass through A?

369. Can we combine Sue's and Barry's answers in a way such that Sue's base is multiplied by Barry's altitude and vice versa?

370. Consider the reflection of A over $\overleftrightarrow{BD}$.

371. Connect P to the midpoint of $\overline{MN}$. How long is this new segment? (Remember, $\triangle MPN$ is a right triangle!)

372. Prove that $A'B/A'C = AB/AC$. What happens if you add 1 to both sides of that?

373. What is $[BFC]/[ADF]$?

374. At how many different points can two circles meet? How about two lines? How about a line and a circle?

375. Let the medians meet at G. What do we know about XG and YG?

376. What is $[ABC]/[ACD]$? What is $[ABC] + [ACD]$?

377. Angle bisectors and side lengths. Try the Angle Bisector Theorem.

378. Let X be the point where $\overline{OP}$ meets $\overline{YZ}$. In terms of our radii, what is OX/XP?

379. Parallel lines mean similar triangles!

380. Draw altitudes from P to the sides of $ABCD$.

381. Draw altitudes from P to each of the sides of the rectangle.

382. Find a couple pairs of similar triangles.

383. In any given hour, how is the total amount of water that flows through the trapezoid related to the combined total amount of water that flows through the holes?

384. We have a $30°$ angle. Build a useful 30-60-90 triangle by drawing the altitude from B to $\overline{AC}$.

385. Write the correct Power of a Point relationship. Combine it with Jake's accidentally correct equation.

386. What kind of quadrilateral is $ACNM$? (Don't just guess – you have to prove it!)

387. We know the power of point P.

388. What piece of information did you not use in the first part? Can you use this along with the result of the first part to find similar triangles?

389. Connect the center of the sphere to the centers of the circles of which the wires are arcs.

390. What is $[QPX]/[WPX]$?

391. Can we still cut the quadrilateral into two triangles?

392. Using your two equations, try to find two different approaches to get the solution. First, try guessing integer solutions (use the equation for xy first!) or use your knowledge of Pythagorean triples. Then, for an alternative solution, expand $(x + y)^2$ and use your equations.

393. What is $(AF)(AD)$?

394. Let the parallelogram be $ABCD$. Draw altitudes from A and B to $\overleftrightarrow{CD}$. See a rectangle?

395. (For $XY > 2YZ$.) Start a new diagram. We want $XY > 2YZ$, so if we pick a point Q on $\overline{XY}$ such that $YQ = YZ$, all we have left to show is that $XQ > YZ$.

396. What inequality can we use regarding the sides of a triangle in order to show that the triangle is acute?

397. What did you learn in the previous problem?

398. Show that in each step after the first step, the number of triangles added is 4 times the number of triangles added in the previous step. How much smaller is each triangle added in a given step than the triangles added in the previous step?

399. Mark equal angles and find lengths equal to DZ and lengths equal to CY.

400. Do we know the sum of the interior angles of the figure?

401. Find $\angle CAB$, $\angle ACD$, and $\angle BCD$ in terms of $\angle B$.

402. What is the length of the altitude from O to face ABC?

403. Find CF first.

404. What kind of triangle is $\triangle EFG$?

405. Let the semicircles have centers Q and R, and let S be the point besides O where they meet. Draw $\overline{QS}$ and $\overline{SR}$.

406. Can you build a useful right triangle with $\overline{EI}$ as hypotenuse?

407. What are segments $\overline{BP}$ and $\overline{MN}$ in $\triangle AMB$?

408. Look back at the tactic we took for a similar problem in the text.

409. Find OS a few different ways.

410. The remaining two pieces of the region enclosed by the bold line after you draw the square are triangles. Can you find any triangles congruent to these?

411. Find AC and the length of the altitude from B to $\overline{AC}$.

412. How is $[XAD] + [XBC]$ related to the area of $ABCD$? What about $[XCD] - [XAB]$?

413. Can you think of a right triangle that must be similar to a triangle that satisfies the conditions of the problem? (Make sure you include why the triangles are similar!)

414. The ratio of the sides of one pentagon to corresponding sides of another is always the same. Each of the angles of one pentagon equals the corresponding angles of the other. What does this suggest about the two pentagons?

415. Can you sketch a Power of a Point-like diagram in which we have segments of lengths a, b, and $\sqrt{ab}$? Now use your straightedge and compass to recreate this diagram.

416. Draw radii from the centers of the circles nearest A and C to the points where these circles are tangent to the square.

417. Can you find some congruent triangles?

418. Draw the perpendicular from N to $\overline{EF}$ and connect G to P.

419. What is the image of $\overline{BN}$ upon rotation about the center of the square?

420. Use similar triangles to find the distance from E to each vertex of the trapezoid.

421. What kind of triangles are $\triangle ABD$ and $\triangle BCD$? Mark all the angles you can find.

422. Are there other diagonals equal in length to $\overline{BH}$ that are easier to compare to AE?