Deductive Reasoning

Is the process of drawing conclusions from given information by using rules of logic. You must be able to support any statement you make.

"Be careful of that wasp: it might sting."

This is deductive reasoning because wasps as a class have stingers; therefore, each and every wasp must have a stinger. We do not need to analyze each and every wasp to understand if it can or cannot sting.

(By the way, there are over 300 species of wasps)

Syllogism

"Every triangle has three sides."
"The figure I drew is a triangle."
"Therefore my figure must have three sides."

"Every member of Congress is a woman."
"Mel Gibson is a member of Congress."
"Therefore Mel Gibson must be a woman."

Deductively, this reasoning is absolutely fine; it is *valid*. An argument such as this is *valid* despite both statements being obviously false. However, the argument is not *sound*. It is not *sound* because both premises are false.

For an argument to be deductively sound the premises must also be true.

What is the difference between Inductive and Deductive Reasoning?

Isaac Newton watches an apple fall to the ground and he induces his theory of gravity. Adams and LeVerrier applied Newton's gravitational theory to deduce the mass, position and orbit of Neptune from that of Uranus.

Counterexample: Any example you can provide that shows a statement to be false.

Statement	Counterexample
1. All birds can fly.	
2. Only penguins are ferocious polar animals.	
3. All four sided-figures are squares.	
4. For all integers x, $x^2 > 0$.	
5. For all real numbers a and b, no solution exists for $a + b = a * b$.	
6. For any real numbers, $(x - y)^2 = x^2 - y^2$.	

Conditional Statements
H Geometry *"If…then…"*

If p, then q.

hypothesis conclusion

If Skylar finds a $20, then I'll take you to the movies.

hypothesis conclusion

Consider the four possibilities:

1. *Skylar finds $20 and takes you to the movies*. She keeps her promise; her statement is **true.**

2. *Skylar finds $20 and does not take you to the movies*.
   She broke her promise; her statement is **false**.

3. *Skylar does not find $20 but still takes you to the movies*.
   She keeps her promise: her statement is **true**.

4. *Skylar does not find $20 and does not take you to the movies*.
   She has not broken her promise; her statement is **true**.

From the previous slide we can make a truth table for conditional statements, “If p, then q”

p	**q**	**p -> q**
T	T	T
T	F	F
F	T	T
F	F	T

Summary of Related Conditionals

Given Conditional	If p, then q	p -> q
Converse	If q, then p	q -> p
Inverse	If not p, then not q	~p -> ~q
Contrapositive	If not q, then not p	~q -> ~p

Write the converse, inverse, and contrapositive of each given conditional statement.

1. Given Conditional: *If two angles are adjacent, then the two angles have the same vertex.*

2. Given Conditional: *If today is Tuesday, then tomorrow is Wednesday.*

Write the converse, inverse, and contrapositive of each given conditional statement.

3. Given Conditional: *If $x > 5$, then $x^2 > 25$.*

4. Given Conditional*: If a polygon is a square, then it is a rectangle.*

Venn diagrams were created by the famous logistician John Venn in the 1880s. He entered Gonville and Caius College Cambridge (Cambridge University) in 1853 basically ignorant of literature, yet was awarded a math scholarship and finished in the top 6 of his graduating math class. At Cambridge, Venn graduated and was an ordained a priest in 1857 and published famous works of logic in the 1880s. His Venn Diagrams were first introduced in 1881 through his book <u>Symbolic Logic</u>. You can say he is the father of modern statistics.

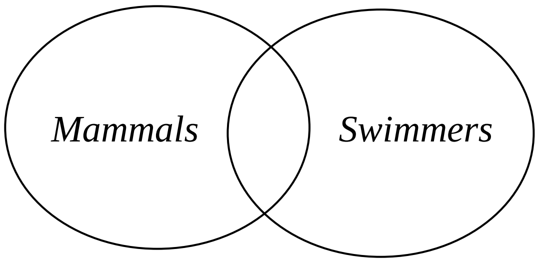
He was also an accomplished machinist, botanist, mountain climber, and linguist. He developed a mechanical cricket bowler that is blew away the Australian team that visited Cambridge.

His most famous contribution to modern math is the Venn Diagram which illustrate logical relationships among data in a picture diagram.

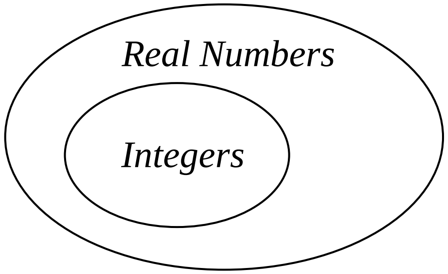
"All, Some or No" Venn Diagrams

Please write All, Some, Or No for the following Venn Diagrams.

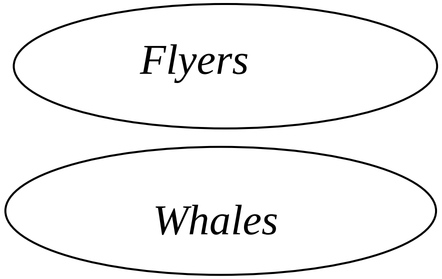
***Some* mammals are swimmers.**



***All* integers are real numbers.**

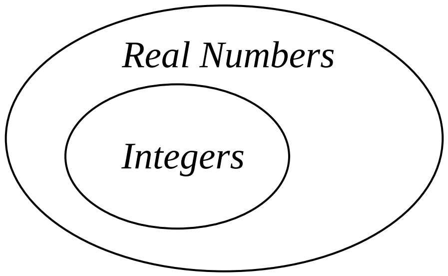


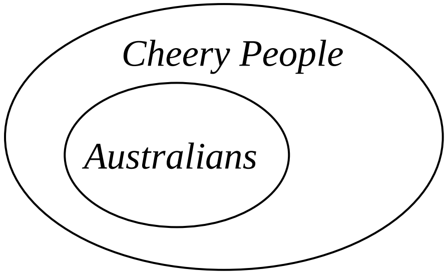
***No* whales are flyers.**



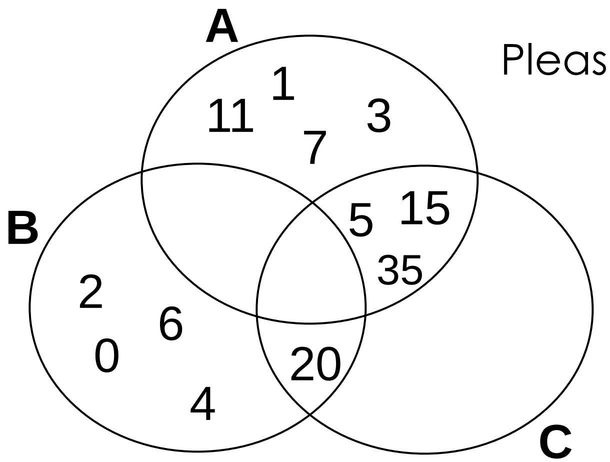
Writing Conditional Statements from Venn Diagrams and vice versa.

(i) Please write the conditional statement given the Venn Diagram.





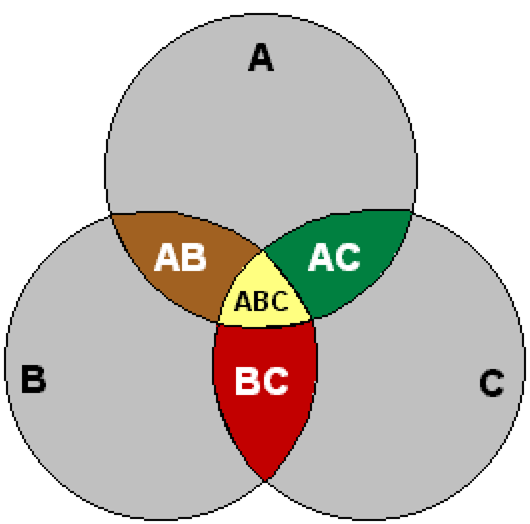
(ii) Draw a Venn Diagram from the statement:
"All Americans are patriotric."

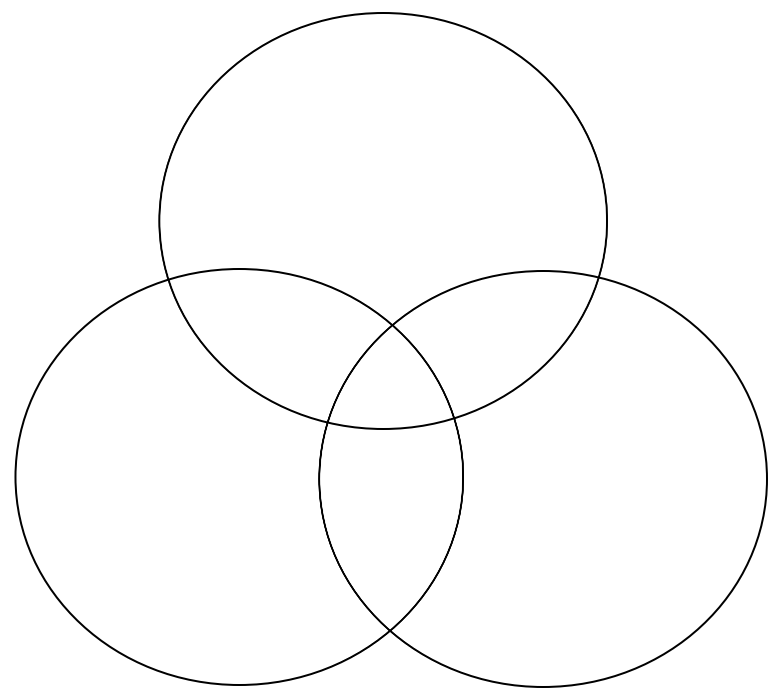


Please describe each set A, B and C.

At Rosemead High School, 94 students take biology, 86 take ethnic studies, And 95 take geometry. Thirty-seven students take both biology and geometry, 43 take geometry and ethnic studies, and 42 take ethnic studies and biology. Twenty-eight students take all three subjects.

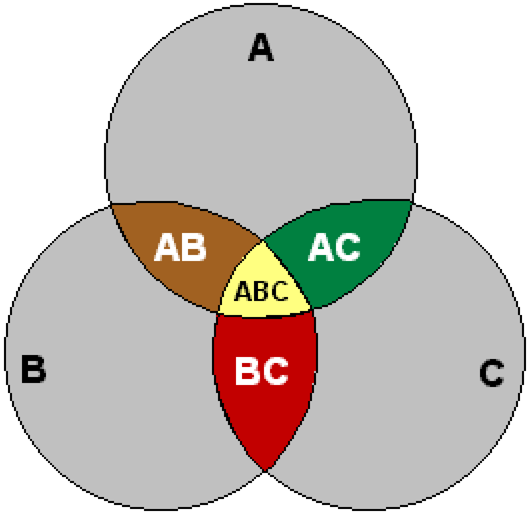
a) Draw a Venn Diagram to illustrate this problem. How many students take geometry but not ethnic studies or biology? What is the probability that a student at random will be taking all three of the courses?

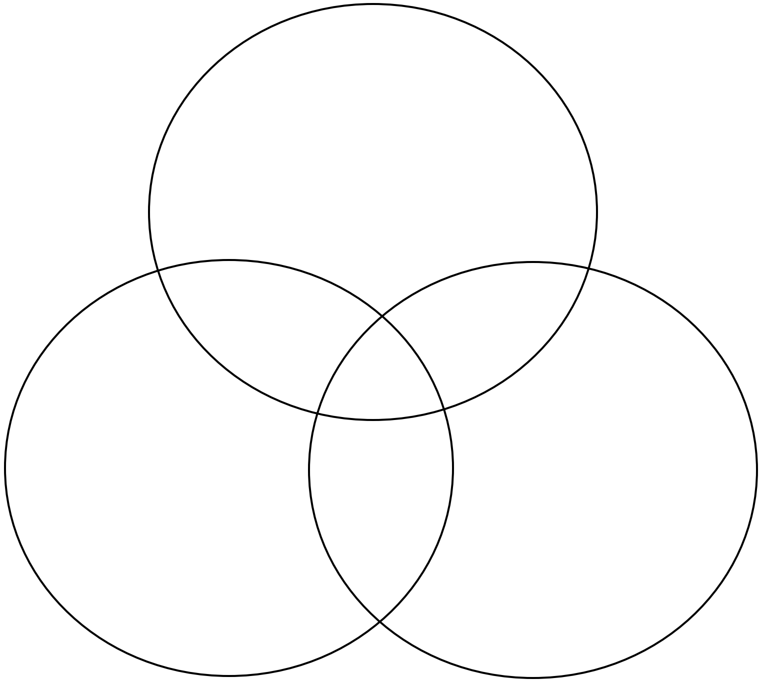




At Rosemead High School, 94 students play football, 86 play soccer, and 65 play golf. Thirty-one students play both football and soccer, 29 play soccer and golf, and 34 play football and golf. 15 students play all three sports.

a) Draw a Venn Diagram to illustrate this problem. How many students play football but not soccer or golf? What is the probability that a student at random will be playing all three of the sports?





We can link two logical statements by conjunctions **AND; OR**.

If we link two statements with **AND:** both statements must be TRUE for the whole statement to be true.

Mr. Messick loves geometry and his students love it too.

If we link two statements with **OR**; only one of the two statements must be true for the whole statement to be true.

Mr. Messick won a gold medal at the 1992 Olympics or Mr. Messick teaches geometry.

What is wrong with the following examples?

Ex. 1 I know SpongeBob is older than Patrick because Patrick is younger than SpongeBob. And Patrick must be younger than SpongeBob because SpongeBob is older than Patrick.

Ex. 2 Skiing and skating are similar because in each sport you fasten runners to your feet. Since skiers wax their skis, skaters must wax their skates.

Ex. 3 All cheerleaders have pleasant personalities.
The Jones girls are not cheerleaders.
Therefore the Jones girls do not have pleasant personalities.

Ex. 4 All good athletes are strong competitors.
The members of this club are strong competitors.
Therefore the members of this club are good athletes.

Biconditional Statement: *A single true statement that combines a true Conditional Statement and its true Converse.*

Ex. 3 Conditional: If N is even, then N + 1 is odd.

Converse:

Biconditional:

Ex. 4 Conditional: If a figure has three sides, then it is a triangle.

Converse:

Biconditional:

***Biconditional Statement**: A single true statement that combines a true Conditional Statement and its true Converse.*

Ex. 3 Conditional: If the sum of the measures of two angles is 180˚, then the two angles are supplementary.

Converse: If two angles are supplmentary, then the sum of the measures of the two angles is 180˚.

Biconditional: Two angles are supplmentary if and only if the sum of the measures of the two angles is 180˚.

Ex. 4 Conditional: If a figure is a square, then it is a rectangle.

Converse: If a figure is a rectangle, then it is a square.

Biconditional:

Biconditional Statement: *A single true statement that combines a true Conditional Statement and its true Converse.*

Ex. 3 Conditional: If N is even, then N + 1 is odd.

Converse:

Biconditional:

Ex. 4 Conditional: If a figure has three sides, then it is a triangle.

Converse:

Biconditional:

Law of Detatchment: *If the hypothesis of a true conditional is true, then the conclusion is true.*

Ex. 1 Given: If a student gets an A on a final exam, then the student will pass the course.
Henry got an A on his math final exam.

Conclude:

Ex. 2 Given: If a two angles are adjacent, then they share a common vertex.
Angle 1 and Angle 2 share a common vertex.

Conclude:

Law of Syllogism: *Stating a conclusion from (2) true conditional statements when the conclusion of one statement is the hypothesis of the other statement.*

Ex. 1 Given: If a figure is a square, then the figure is a rectangle.
If a figure is a rectangle, then the figure has four sides.

Conclude:

Ex. 2 Given: If you take algebra, then you like math.
If you take geometry, then you like math.

Conclude:

Ex. 3 Given: If a whole number ends in 6, the it is divisible by 2.
If a whole number ends in 4, then it is divisible by 2.

Conclude:

Property	Definition	Addition	Multiplication
Commutative	Changing the order of the number will not change the result.	a + b = b + a Ex: 2 + 3 = 3 + 2 = 5	a * b = b * a Ex: 2 * 3 = 3 * 2 = 6
Associative	Changing the grouping of the numbers will not change the result.	a + (b + c) = (a + b) + c Ex: 1 + (2 + 3) = (1 + 2) + 3 = 6	a * (b * c) = (a * b) * c Ex: 1* (2 * 3) = (1 * 2) * 3 = 6
Identity	**Zero** and **one** preserves identities under addition or multiplication respectively.	a + 0 = 0 + a = a Ex: 2 + 0 = 0 + 2 = 2	1 * a = a * 1 = a Ex: 1 * 2 = 2 * 1 = 2
Inverse	For each real number a, there exist a unique number - a and 1/a for additive or multiplicative inverse.	a + (- a) = 0 Ex: 2 + (- 2) = 0	a * 1/a = 1 Ex: 2 * ½ = 1
Distributive	Multiplication distributes over addition. a(b + c) = ab + ac	—	—

H Geometry